Tutorial Sheet 5: Graph Theory

(1) Draw these graphs.
$(a)K_7$ $(b)K_{1,8}$ $c)K_{4,4}$ $(d)C_7$ (e) W_7 $(f)Q_4$.

(2) For which values of n are these graphs bipartite?
$(a)K_n$ $(b)C_n$ $(c)W_n$ $(d)Q_n$.

(3) Let n be a positive integer. Show that a subgraph induced by a nonempty subset of the vertex set of K_n is a complete graph.

(4) How many subgraphs with at least one vertex does K_3 have?

(5) For which values of n are these graphs regular?
$a)K_n$ $(b)C_n$ $(c)W_n$ $(d)Q_n$.

(6) Find an adjacency matrix for each of these graphs.
$(a)K_n$ $(b)C_n$ $(c)W_n$ $(d)K_{m,n}$ $(e)Q_n$.

(7) Find a self-complementary simple graph with five vertices.

(8) How many nonisomorphic simple graphs are there with five vertices and three edges?

(9) What is the product of the incidence matrix and its transpose for an undirected graph?

(10) Show that every connected graph with n vertices has at least $n-1$ edges.

(11) Show that if a connected simple graph G is the union of the graphs G_1 and G_2, then G_1 and G_2 have at least one common vertex.

(12) Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.

(13) For which values of n do these graphs have an Euler circuit?
$(a)K_n$ $(b)C_n$ $(c)W_n$ $(d)Q_n$.

(14) Show that a bipartite graph with an odd number of vertices does not have a Hamilton circuit.

(15) Show that K_5 is nonplanar.

(16) Suppose that a connected bipartite planar simple graph has e edges and v vertices. Show that $e \leq 2v-4$ if $v \geq 3$.

(17) Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?